USE OF MICROMETEOROLOGICAL DATA IN THE COMPUTATION OF THE FRICTION VELOCITY

JAMES L. JOHNSTON

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James I. Johnston

Lieutenant, Unites States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
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ABSTRACT

This investigation was undertaken to determine the feasibility of computing the friction velocity from available micrometeorological data. This could be of great assistance in finding the wind velocity profile by Deacon's equation and the eddy shearing stress in the lower portion of the friction layer.

This investigation was carried out at the U. S. Naval Postgraduate School, Monterey, California, during the period February 1959 - May 1959, in partial fulfillment of the requirements for the degree of Master of Science in Meteorology.

The author wishes to acknowledge the assistance and guidance given him by Professor F. L. Martin. He also wishes to express his appreciation to his wife for her patience and understanding during the many nights spent by him with the electronic computor.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

TABLE OF SYMBOLS AND ABBREVIATIONS

Letters:

^{o}C	degrees Centigrade
Ri	Richardson's number
T	absolute temperature
g	acceleration due to gravity
k	von Karman's constant
ℓ	mixing length
ln	natural logarithm
mph	miles per hour
t	time
u	horizontal wind component
u_*	friction velocity
w^*	vertical mixing velocity
x	characteristic dimensionless function
z	height
z_o	roughness parameter
β	stability parameter
θ	potential temperature
ρ	density of air
τ	eddy shearing stress

Subscripts:

a	adiabatic
avg	average for entire run
m	mean for layer considered at a given time
numbers	height or layer considered

Superscripts:

bar	weighted average of all layers at a given time

Mathematical Symbols:

$=$	equal to
$\leq$	less than or equal to
$\neq$	unequal to
$\lvert\ \rvert$	absolute value
$\frac{\partial}{\partial z}$	partial derivative with respect to height
Δ	differences
$\sum_{i=1}^{N}$	summation of all value from i = 1 to i = N

1. Introduction

The friction velocity, a stability parameter, and a term related to the Richardson number were computed using micrometeorological data. The equations used for the computation of these three parameters are derived. These equations are unique in that they combine the different theories of two well-known investigators of surface layer turbulence into an equation for the friction velocity and an equation for the wind profile in the surface layer.

The micrometeorological data used in this investigation is discussed, as well as the instruments used in obtaining the data and the observational errors. A theory is presented on errors of aerated temperatures close to the ground.

After modifying the derived equations into a form suitable for using with an electronic digital computor, the so-called "constants" in these equations are discussed. The advantages and disadvantages of using an electronic computor for this type of an investigation are then summarized.

Various figures and graphs are used to illustrate the diurnal variation as well as the possible variation with height as determined from this investigation. A comparison of these results with those of previous investigators is made, indicating close agreement in most cases.

2. Derivations

Deacon [2], in the derivation of his wind profile equation, as well as most other well-known investigators in the field of surface layer turbulence, considered only two virtual turbulence parameters - namely, the mixing length ℓ and the friction velocity u_*, where

$$u_* = \sqrt{\frac{\tau}{\rho}}$$

Here τ is the eddy stress and ρ is the air density. Lettau [10] introduces a third parameter, the vertical mixing velocity $\overset{*}{w}$, that is quite important if nonadiabatic rather than neutral conditions are investigated. He also has a term which he calls the friction velocity $\overset{*}{u}$, where

$$\overset{*}{u}\overset{*}{w} = \frac{\tau}{\rho}$$

However, when the friction velocity is mentioned in this paper it will refer to the first definition.

The surface layer is defined as that layer, immediately above the earth's surface, in which the eddy stress is constant with elevation throughout the layer. Eqs. (1-9) are descriptive of this layer. The notation[1] is that employed by Lettau in [10].

In a neutral surface layer the equations below are applicable.

$$\overset{*}{u}_a = \overset{*}{w}_a \tag{1}$$

[1]Throughout this paper u will be understood to mean the wind velocity at some level averaged over some increment of time - say six minutes.

$$\ell_a = k(z + z_o) \qquad k = \text{von Karman's constant} \quad z_o = \text{roughness parameter} \tag{2}$$

$$\overset{*}{u}_a = \ell_a \frac{\partial u_a}{\partial z} \tag{3}$$

$$\overset{*}{u}_a \overset{*}{w}_a = \frac{\tau_a}{\rho} \tag{4}$$

$$u_{*_a} = \sqrt{\frac{\tau_a}{\rho}} = \overset{*}{u}_a \tag{5}$$

It should be noted that in Eqs. (1-5), $\overset{*}{u}_a$, $\overset{*}{w}_a$, and τ_a are constant with elevation in the neutral surface layer.

In a non-neutral surface layer the next four equations are used.

$$\overset{*}{u} \neq \overset{*}{w} \tag{6}$$

$$\overset{*}{u}\overset{*}{w} = \frac{\tau}{\rho} \tag{7}$$

$$u_* = \sqrt{\frac{\tau}{\rho}} \tag{8}$$

$$\tau_a \neq \tau \tag{9}$$

In [10], Lettau derives a relationship between $\overset{*}{w}$ and $\overset{*}{w}_a$ which is of fundamental importance in the development of the remainder of this paper. Two basic assumptions are necessary in the derivation of Lettau's relationship:

1. The ratio $\overset{*}{w}/\ell$ does not depend upon variations of thermal stratification, so that

$$\frac{\overset{*}{w}}{\ell} = \frac{\overset{*}{w}_a}{\ell_a} \tag{10}$$

Eq. (10) indicates that the vertical mixing length is changed in exactly the same ratio as the vertical mixing velocity if the thermal stratification varies.

2. The second assumption is that the buoyant acceleration influences only the vertical parameters of turbulence. This assumption is necessary in order to fix absolutely the variation of mixing velocity and mixing length with thermal stratification. In an adiabatic atmosphere the term $\overset{*}{w}_a{}^2/\ell_a$ can be considered to represent the vertical component of a turbulent acceleration. Thus we have the vertical acceleration equation:

$$\frac{\overset{*}{w}{}^2}{\ell} = \frac{\overset{*}{w}_a{}^2}{\ell_a} - \frac{g\frac{\partial\theta}{\partial Z}\ell}{T_m} \tag{11}$$

where g = acceleration due to gravity, $\partial\theta/\partial Z$ = change in potential temperature with respect to height, and T_m = mean absolute temperature averaged with respect to height in the layer considered. Eq. (11) indicates that the mean vertical turbulent acceleration in a non-adiabatic surface layer equals the adiabatic acceleration plus the buoyant acceleration.

Upon multiplying Eq. (11) by $\ell/\overset{*}{w}_a{}^2$

$$\frac{\overset{*}{w}{}^2}{\overset{*}{w}_a{}^2} = \frac{\ell}{\ell_a} - \frac{g\frac{\partial\theta}{\partial Z}\ell^2}{T_m\overset{*}{w}_a{}^2}$$

but from Eq. (10), $\overset{*}{w}{}^2/\overset{*}{w}_a{}^2 = \ell^2/\ell_a^2$, so upon substituting

$$\frac{\ell^2}{\ell_a^2} = \frac{\ell}{\ell_a} - \frac{g\frac{\partial\theta}{\partial Z}\ell^2}{T_m\overset{*}{w}_a{}^2}$$

multiplying through by ℓ_a^2/ℓ^2 gives

$$1 = \frac{\ell_a}{\ell} - \frac{g \frac{\partial\theta}{\partial Z} \ell_a^2}{T_m \overset{*}{w}_a^2}$$

or,
$$\frac{\ell_a}{\ell} - 1 = \frac{g \frac{\partial\theta}{\partial Z} \ell_a^2}{T_m \overset{*}{w}_a^2}$$

Let
$$x = \frac{g \frac{\partial\theta}{\partial Z} \ell_a^2}{T_m \overset{*}{w}_a^2} \tag{12}$$

Therefore,
$$x = \frac{\ell_a}{\ell} - 1 \tag{13}$$

Thus we finish with Lettau's derivation by putting ℓ/ℓ_a in terms of x,

$$\frac{\ell}{\ell_a} = \frac{1}{(1+x)} \tag{14}$$

The parameter x is important in the computations to be made in this paper.

The next derivation was originally developed by Martin [12] combining Deacon's wind profile with Lettau's vertical mixing velocity.

Starting with Deacon's Equation

$$\frac{\partial u}{\partial Z} = \frac{u_*}{k Z_o}\left(\frac{Z + Z_o}{Z_o}\right)^{-\beta} \tag{15}$$

where β, as defined by Deacon, is a decreasing function of the Richardson number. Combining Eq. (7) and (8), one obtains

$$u_* = \sqrt{\overset{*}{u}\,\overset{*}{w}}$$

where $\overset{*}{u} = \ell \dfrac{\partial u}{\partial z}$

and $\overset{*}{w} = \dfrac{\ell}{\ell_2}\,\overset{*}{w}_2$

So that $$u_* = \sqrt{\frac{\ell^2}{\ell_2}\,\overset{*}{w}_2\,\frac{\partial u}{\partial z}}$$

Upon substituting into Eq. (15)

$$\frac{\partial u}{\partial z} = \frac{\sqrt{\frac{\ell^2}{\ell_2}\,\overset{*}{w}_2\,\frac{\partial u}{\partial z}}}{kZ_0}\left(\frac{Z+Z_0}{Z_0}\right)^{-\beta}$$

But from Eq. (14) $\dfrac{\ell}{\ell_2} = \dfrac{1}{(1+X)}$, so that after multiplying inside the radical by ℓ_2/ℓ_2

$$\frac{\partial u}{\partial z} = \frac{\sqrt{\frac{\ell_2}{(1+X)}\,\frac{\overset{*}{w}_2}{(1+X)}}\;\sqrt{\frac{\partial u}{\partial z}}}{kZ_0}\left(\frac{Z+Z_0}{Z_0}\right)^{-\beta}$$

Dividing through by $\sqrt{\dfrac{\partial u}{\partial z}}$

$$\sqrt{\frac{\partial u}{\partial z}} = \frac{\sqrt{\ell_2\,\overset{*}{w}_2}}{kZ_0(1+X)}\left(\frac{Z+Z_0}{Z_0}\right)^{-\beta}$$

Squaring out:

$$\frac{\partial u}{\partial z} = \frac{\ell_a \overset{*}{w}_a}{k^2 z_o^2 (1+x)^2} \left(\frac{z + z_o}{z_o} \right)^{-2\beta}$$

But from Eq. (2) $\ell_a = k(z + z_o)$, so that

$$\frac{\partial u}{\partial z} = \frac{k \overset{*}{w}_a (z+z_o)}{k^2 z_o^2 (1+x)^2} \left(\frac{z + z_o}{z_o} \right)^{-2\beta}$$

or,

$$\frac{\partial u}{\partial z} = \frac{\overset{*}{w}_a}{k z_o (1+x)^2} \left(\frac{z+z_o}{z_o} \right) \left(\frac{z + z_o}{z_o} \right)^{-2\beta}$$

Further combining of terms gives the equation in its final form. Namely,

$$\frac{\partial u}{\partial z} = \frac{\overset{*}{w}_a}{k z_o (1+x)^2} \left(\frac{z + z_o}{z_o} \right)^{1-2\beta} \tag{16}$$

Thus Martin has eliminated u_* from Deacon's Equation - a parameter that is difficult to determine with accuracy.

Rewriting Eq. (16) in the form

$$\frac{\partial u}{\partial z} = \frac{\overset{*}{w}_a}{(1+x)^2} \left(\frac{z + z_o}{z_o} \right)^{1-\beta} \left[\frac{1}{k z_o} \left(\frac{z + z_o}{z_o} \right)^{-\beta} \right] \tag{17}$$

and comparing it with Eq. (15), one can easily see that an equation for the friction velocity has evolved:

$$u_* = \frac{\overset{*}{w_o}}{(1+x)^2}\left(\frac{z+z_o}{z_o}\right)^{1-\beta} \tag{18}$$

Using this equation to find u_* one needs only to know the adiabatic mixing velocity, roughness parameter, vertical temperature gradient, and the stability parameter.

This completes the necessary derivations. The remainder of the paper deals with the solution of Eq. (2) for x, Eq. (16) for β, and Eq. (18) for u_*, using available micrometeorological data.

3. Data

Immediately upon deciding to conduct this investigation into micrometeorology a search was initiated to find suitable data. It soon became apparent that there was a scarcity of micrometeorological data of the caliber necessary to conduct individual research into surface layer turbulence. Several sets of available published data had to be discarded because one or another necessary parameter had not been observed, the period of observation was too short, or the heights of the observations were not at suitable locations.

The best data that could be found within a reasonable time was that by Gerhardt [5] in 1949 and by Gerhardt [6] in 1950. The 1949 data that was of interest in this investigation consisted of wind observations at 12', 41', and 91'; aerated temperatures at 3', 6', 12', 20', 35', 55', 80', and 110'; non-aerated temperatures at 1", 6", 18", 3', 5', 9', and 15'. The 1950 data included in addition to the above, wind at 3' and non-aerated temperatures at 23', 33', 45', 60', 78', and 99'.

The information was gathered during runs of from 24 to 44 hours for different seasons of the year, with an average run being about 30 hours. Observations were taken every half hour during a run, with the final value for that time and height being an average of five readings taken over a six-minute interval.

Jehn [9] and Gerhardt [7] give a detailed explanation of the instruments used and their accuracy, the topography of the micrometeorological site area, and the vegetation at the site and surrounding fields. The wind measurements were taken with Bendix-Friez Aerovanes to the nearest mph. The errors involved were less than 1 mph for wind speeds in the range 10-50 mph, increasing to nearly 2 mph after 13 months of operation. For wind speeds less than 10 mph the errors were slightly greater, and readings

below 3 mph were unsatisfactory.

The wind measurements, because of their inaccuracy and the spacing of instruments, were a severe handicap in this investigation. However, it was hoped that by analyzing a large number of observations, the errors would be somewhat minimized.

The temperatures were obtained from calibrated rod-shaped thermistors (ceramic resistors), about 0.75 inches long and 0.03 inches in diameter. All thermistors were fitted to a single calibration curve giving an accuracy over the range of $0^{o}C$ to $50^{o}C$ of $\pm$ $0.1^{o}C$. The aerated thermistors were mounted in anti-radiation housings, being ventilated by a conventional motor and exhaust fan arrangement at the top of the housing. The non-aerated thermistors were coated with an anti-radiation paint and, in later runs, were shielded from both solar and terrestrial radiation without reducing the natural ventilation available.

Because of all the precautions taken in calibration and shielding, one could expect that there would be little or no difference between the aerated and non-aerated temperatures. However, during this investigation it soon became evident that there were large differences (in some cases nearly $1^{o}C$) between the two elements, and in several instances one gave an increase in temperature with height while the other gave a decrease over the same layer. In order to determine which would be best suited for this investigation, a thorough analysis of both sets of temperatures was conducted for two runs. It was found that at elevations greater than five feet the non-aerated temperatures were higher than the aerated both day and night, with the differences usually .2-.$6^{o}C$ for all pairs of thermistors. For elevations lower than five feet, however, it was discovered that the non-aerated thermistors were cooler during the early afternoon and warmer during the early morning, with the differences increasing with decreasing

elevation. This at first seemed puzzling, since it is contrary to what one would intuitively expect -- at least for early afternoon. However, the author believes that this can be reasonably explained.

The magnitude of the vertical temperature gradient decreases rapidly with height near the ground, especially during early morning and afternoon. Now consider the ventilation system drawing in an equal volume of air from both above and below the level z. Because of the difference in temperature gradient the absolute value of the temperature difference between z and $z - \Delta z$ will be greater than that between z and $z + \Delta z$. This difference will be greatest during early morning and early afternoon, and thus the aerated temperature will be warmer than the true temperature at z during the afternoon and cooler in the morning.

Because of this effect, it was decided that the non-aerated thermistors would give a more representative vertical gradient. However, of the four runs used in this paper (Runs 8, 10, 16, and 17) only on Run 17 was it possible to use the non-aerated temperatures since on Runs 8 and 10 they were only observed to 15' and on Run 16 the 3' element was broken. Even so, it is felt that greater accuracy in the temperature measurements are not warranted because of the large errors involved with the wind velocities.

Gerhardt, in his final evaluation of the Micrometeorological Research Project (from which the data used in this investigation was obtained), stated:

> The importance of an accurate wind gradient, particularly in the lowest layers, cannot be overestimated; 12 measurement levels below 50 feet would not be excessive. Great care should also be taken to obtain instruments with as nearly identical response as possible. Although thermistors have certain undesirable characteristics, it is believed that the installation as set up was satisfactory, and that accurate and representative air temperatures were obtained.

4. Computations

In this section, the working equations using finite differences in place of derivatives are illustrated. The evaluation of the supposedly "non-variable" parameters is discussed, and the advantages and disadvantages in using an electronic digital computer for this type of computation is pointed out.

The particular working equation used to compute x was obtained by substituting Eq. (2) into Eq. (12) and substituting finite differences in place of the partial derivative $\partial\theta/\partial z$. This results in the working equation

$$x = \left[\frac{g k^2 (z + z_0)^2}{\overset{*}{w}_a (z_2 - z_1)}\right] \frac{\theta_2 - \theta_1}{T_m} \tag{19}$$

The terms inside the bracket are constant for the layer during most of the run, although a new value of $\overset{*}{w}_a$ was selected every 12 hours. It is easily seen from this equation that x is zero for neutral conditions, greater than zero for unstable conditions, and less than zero for stable stratification. Values of $(1+x)^2$ were tabulated.

The next variable to find was β . The partial derivative $\partial u/\partial z$ was first calculated from the wind data using finite differences. Then taking logarithms of both sides of Eq. (16) and solving for β resulted in the following equation.

$$\beta = \frac{1}{2}\left[1 + \frac{\ln k z_0}{\ln \frac{z_0}{z + z_0}} + \frac{\ln \frac{(1+x)^2}{\overset{*}{w}_a} \frac{\partial u}{\partial z}}{\ln \frac{z_0}{z + z_0}}\right] \tag{20}$$

The only variable for a given level is the numerator of the third term inside the brackets.

The final variable to determine for this investigation was the friction velocity. The working equation was developed by taking logarithms of both sides of Eq. (18) and then taking e to the power of the left side and the right side, giving

$$u_* = e^{\ln \frac{\overset{*}{w}_a}{(1+\gamma)^2} + (1-\beta) \ln \frac{z+z_0}{z_0}} \tag{21}$$

This equation looks more difficult to solve than the original equation, but is actually more adaptable to programming on our electronic digital computer.

Prior to solving Eqs. (19), (20), and (21) the "constant" parameters (von Karman's constant k, adiabatic mixing velocity $\overset{*}{w}_a$, and roughness parameter z_o) had to be determined.

According to [8], $\overset{*}{w}_a$ and z_o are easily obtained by plotting the wind speeds against ln z for adiabatic conditions, and drawing a best-fitting straight line. The roughness parameter being the intercept on the ln z axis and the mixing velocity being the slope divided into von Karman's constant, or

$$\overset{*}{w}_a = \frac{k(u_2 - u_1)}{\ln \frac{z_2}{z_1}} \tag{22}$$

The first problem to be solved was that of determining the time, and therefore the wind-sounding, for neutral conditions. All of the layers examined up to 110 feet were of depth not exceeding 10 meters. In layers of such depth, bounded by thermistors, a dry adiabatic lapse rate

corresponds to a vertical temperature difference $|\Delta T| \leq 0.1^{\circ}C$. In such shallow layers, a neutral lapse may be regarded as essentially isothermal. Special emphasis in this regard was given to the temperature difference in the layer 0-15 feet above the ground. Runs 16 and 17 correspond to stabilities nearly neutral for the entire period.

On clear days, a neutral surface layer is considered [8] to occur near sunrise or sunset. This was found to be the case. The actual time of isothermalcy usually occurred between half-hourly wind observations near sunrise. Hence determining appropriate wind values required the averaging of at least two consecutive sets of data centered about the time of isothermalcy. Actually it was decided to average wind speeds for an hour before and after the neutral case in order to reduce round off errors in the reported wind values, since these were given only to the nearest mph. These averaged wind speeds were then plotted against ln z and a straight line fitted. The straight line went through or nearly through all the plotted points. The values of z_o were consistent with those 12 hours earlier or later. The adiabatic mixing velocity w_a^* still varied from the morning to evening adiabatic case. This is, however, to be expected. The adiabatic mixing velocity thus obtained was used for six hours preceding and six hours after the time of neutral stratification.

Suggestions have been made [16], [17] that k is a function of stability. However, most specialists in the field treat k as a constant independent of stability. Values given for it in textbooks and in the literature range from 0.38 to 0.45, with most references giving k as 0.40. This is the value used in this thesis. The particular constant value used for k will not materially affect the results of correlating β with $(1+x)^2$.

It was decided to use the CRA 102-A general purpose automatic digital computer [19] rather than a slide rule or hand computer. This decision was

based on the large number of computations involved and the additional numbers of significant digits possible with the computer.

There are two possible disadvantages in using such a computer, the first being that the data had to be converted from the decimal number system to the octal number system. If the data were whole numbers, such as the wind observations, this presented little difficulty because programs are available for the necessary conversion. However, when involved with mixed numbers, as in the temperature observations, it is necessary to convert the whole number part and the fractional part separately, and then add them together. It was possible, however, after a little experience to rapidly convert the whole numbers mentally, thus necessitating machine conversion of the fractions only. It was found that for a run of 24 hours with wind and temperature at four heights and the mean temperatures of the three layers (a total of 528 numbers) that the data could be converted and put on IBM cards in less than one hour.

The second disadvantage was the writing of the programs for the computer. It is estimated that a total of 20 hours was spent writing the necessary programs (Ri, $\frac{\partial u}{\partial z}$, $(1+x)^2$, β , u_*, and a correlation coefficient program), and another 20 hours spent testing these programs on the computer to make certain they were working properly.

Once this preliminary work was accomplished the remaining work was relatively simple, since the programs were good for <u>any</u> run once a few constants were changed. An example of the time saved will be illustrated. Five friction velocities were computed by hand using logarithms and a slide rule, taking nearly one hour. Using the computer, the friction velocities for one 24-hour run and three layers (244 values) were computed, typed out in both octal and decimal, and IBM cards punched in 15 minutes! Assuming that the time for the hand calculations could be reduced by half, it would

still take over 24 hours or about 100 times as long as the machine.

By having the computer punch IBM cards for all variables it was then an easy matter to determine correlation coefficients between the various parameters. It took the machine 36 seconds to compute the correlation coefficient between two parameters consisting of 48 observations each.

The computer can also plot the information taking a little over a minute to plot 48 values of two variables. This was especially useful because the relatively small range the various parameters had would make it very tedious to plot by hand.

Once a program has been successfully tested, there is little chance of erroneous values. With hand calculations, arithmetical errors may occur at any time.

One last advantage that seems worthy of mentioning is the fact that with all the information on cards, it would be easy for another person to use in an investigation along similar lines - such as in determining the heat flux or the coefficient of eddy viscosity.

5. Results

A discussion of the results of this investigation and a comparison of these results with those of other investigators into surface layer turbulence will now be made. The numerical values of the parameters $(1+x)^2$, β, and u_* for each run may be found in tables in Appendices I, II, III, and IV. It will be seen in these tables that for Runs 16 and 17 three layers were evaluated - 3 to 12 feet, 12 to 41 feet, and 41 to 91 feet. Although Deacon's wind profile is usually considered applicable only to about 41 feet, a higher layer was analysed in order to determine the inconsistencies, if any, from those at lower elevations. Runs 8 and 10 were used, even though it was only possible to get values for the middle layer (due to insufficient data), because of the much greater ranges of stability than for Runs 16 and 17.

The diurnal variation of $(1+x)^2$, β, and u_*, as well as their correlation with one another is illustrated in Fig. 1, where the three parameters for Run 8 (12 to 41 feet) are plotted against time. Run 8 was taken on 27-28 September with clear skies and light to moderate winds prevailing for most of the run. It is seen that $(1+x)^2$ generally follows the diurnal march of temperature, becoming greater than one after sunset and less than one after sunrise, whereas β has the opposite trend. For this particular run β was less than one for only a few hours just prior to midnight, indicating the layer was unstable for the majority of the run.

The inverse relationship between β and $(1+x)^2$ can easily be recognized in Figs. 1, 2, and 3. The curves of Figs. 1 and 6 have been smoothed somewhat using a smoothing equation

$$\beta_{ts} = .25\beta_{t-1} + .50\beta_{t} + .25\beta_{t+1}$$

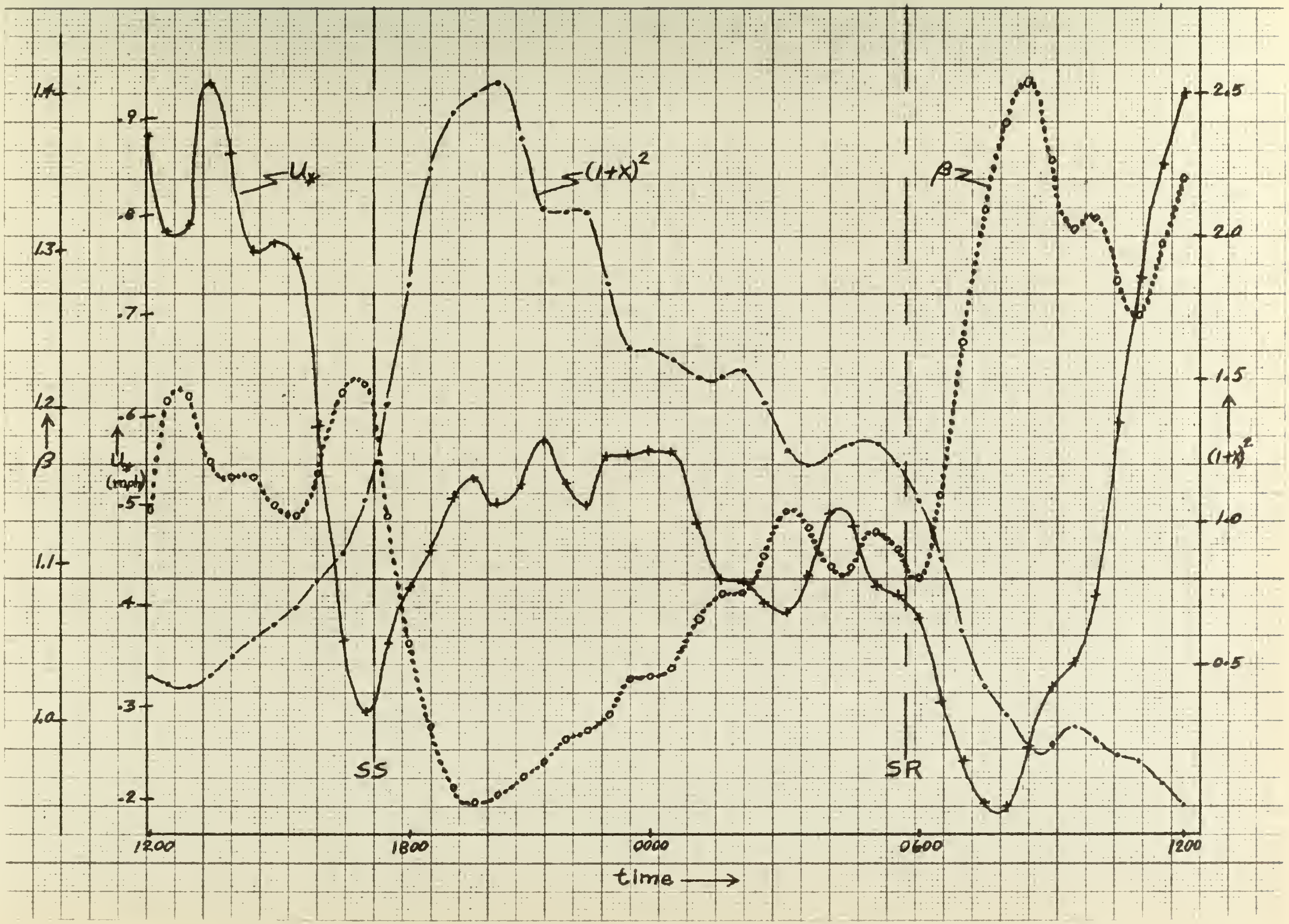

Figure 1. The variation of β, u_*, and $(1+x)^2$ with time of day for Run 8.

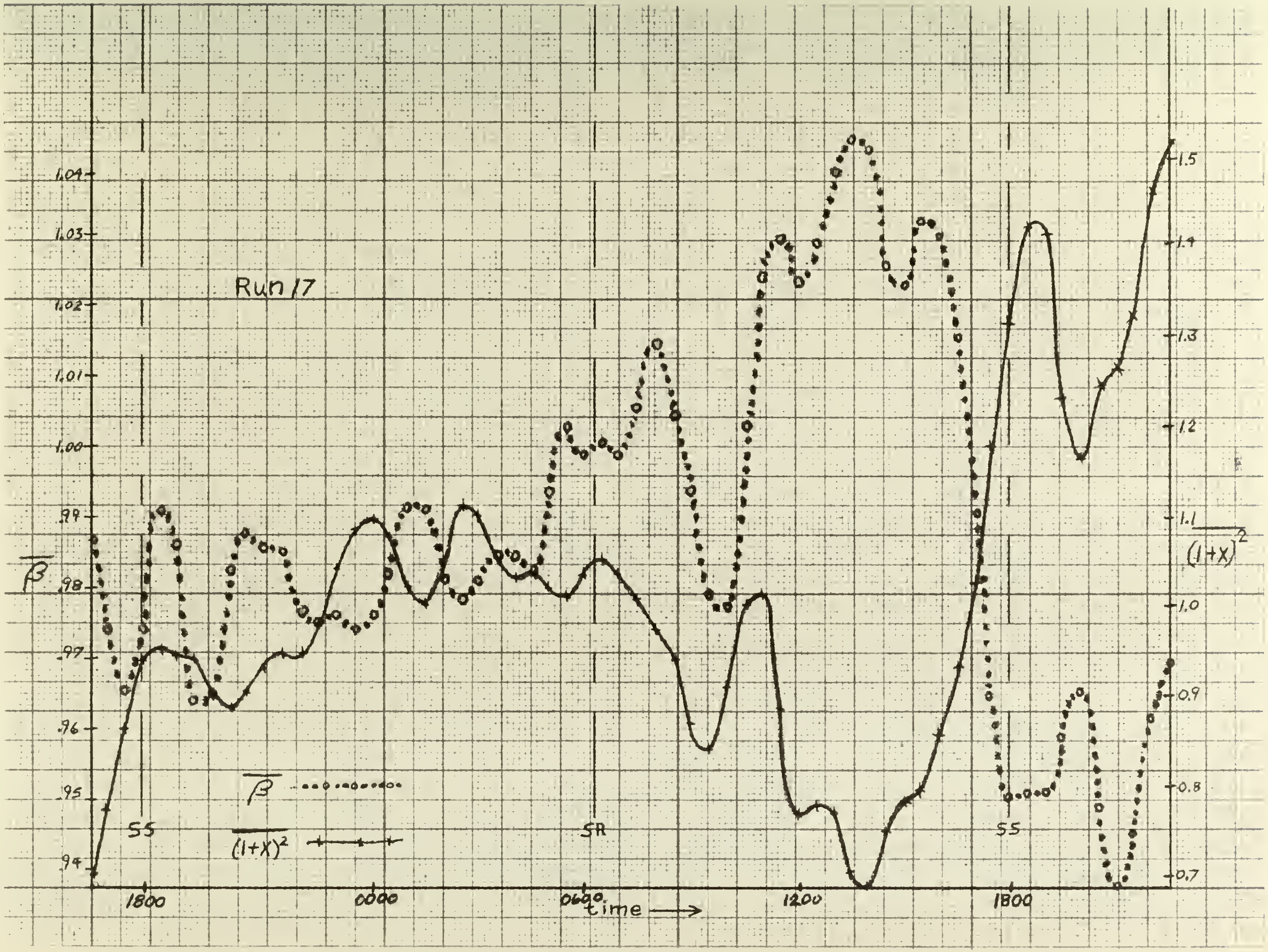

Figure 2. The variation of $\overline{\beta}$ and $\overline{(1+x)^2}$ with time of day for Run 17.

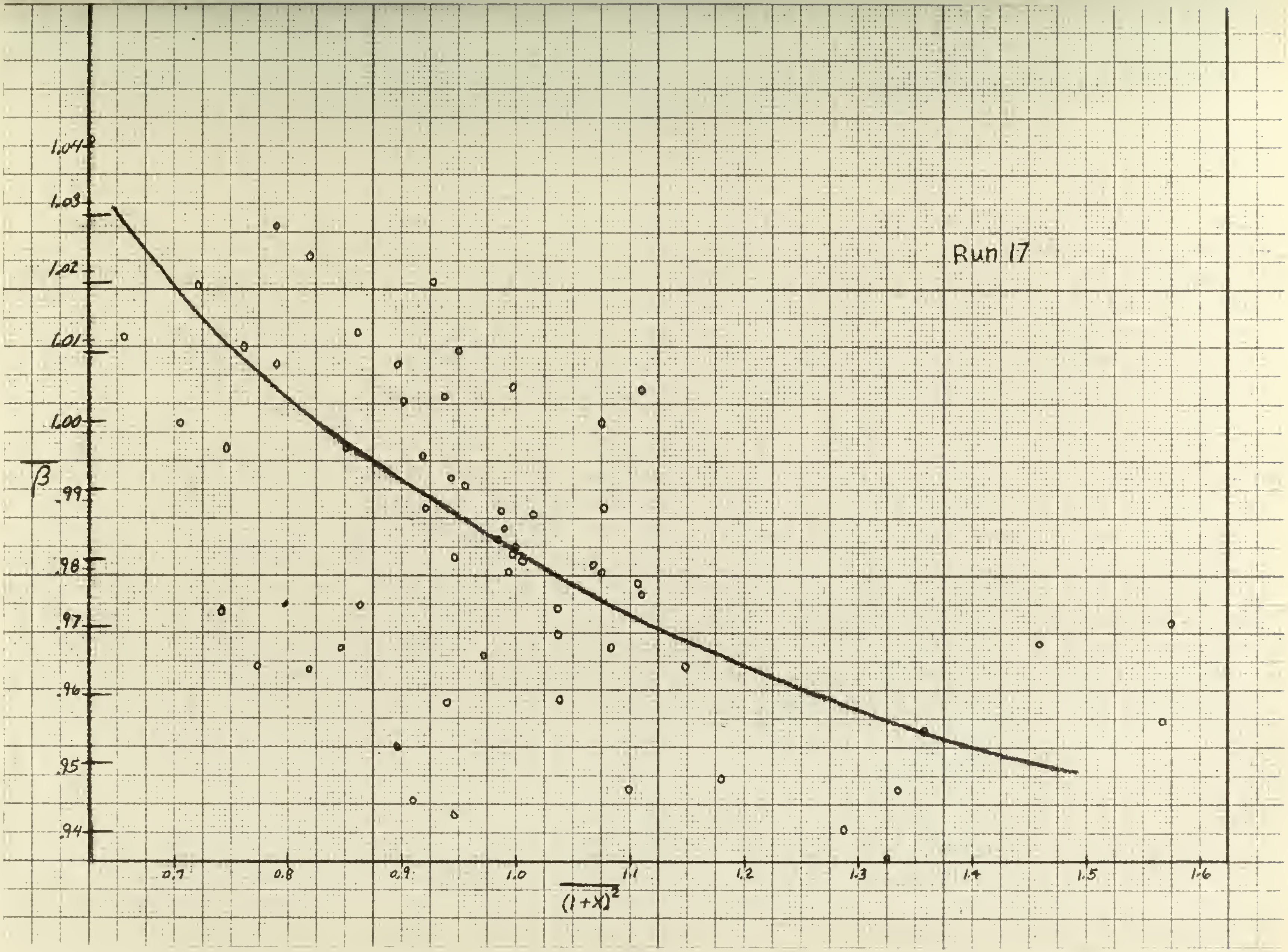

Figure 3. Scatter diagram illustrating the variation of $\overline{\beta}$ with $\overline{(1+x)^2}$ for Run 17.

For a closer examination of the short period fluctuations of β, see the Appendix. A negative correlation between β and $(1+x)^2$ is to be expected since x is directly related to the Richardson number

$$x = \frac{\left(\frac{\partial u}{\partial z}\right)^2}{\left(\frac{\partial u_g}{\partial z}\right)^2} Ri$$

The quantities $\overline{\beta}$ and $\overline{(1+x)^2}$ in Figs. 2 and 3 are weighted means for the entire layer from 3 feet to 91 feet for each observation time of Run 17. The equation used to determine this mean being

$$\overline{\beta} = \frac{1}{88}\left[9\beta_{3-12} + 29\beta_{12-41} + 50\beta_{41-91}\right]$$

and a similar equation for $\overline{(1+x)^2}$. Fig. 3 is a scatter diagram of $\overline{\beta}$ and $\overline{(1+x)^2}$, with the line fitted by eye. The scatter of the points is probably mainly the result of the inaccuracy of the original wind measurements which were given only to the nearest mph. For Run 17 the linear correlation coefficient between $\overline{\beta}$ and $\overline{(1+x)^2}$ of -.54 was determined. This significant negative correlation is considered to indicate the soundness of the Lettau-Martin theory.

In 1948, Sutton [18] deduced that the Richardson number must be independent of height. However, Deacon, in the same year, found from his observations and calculations that Ri varied almost linearly with height. Fig. 4 shows $(1+x)^2$ for each layer plotted against time for Run 16. A decrease of $(1+x)^2$ with height (and thus a decrease of Ri with height) is very apparent in this figure. The average value of $(1+x)^2$ was determined for each layer, using the usual equation to find a mean

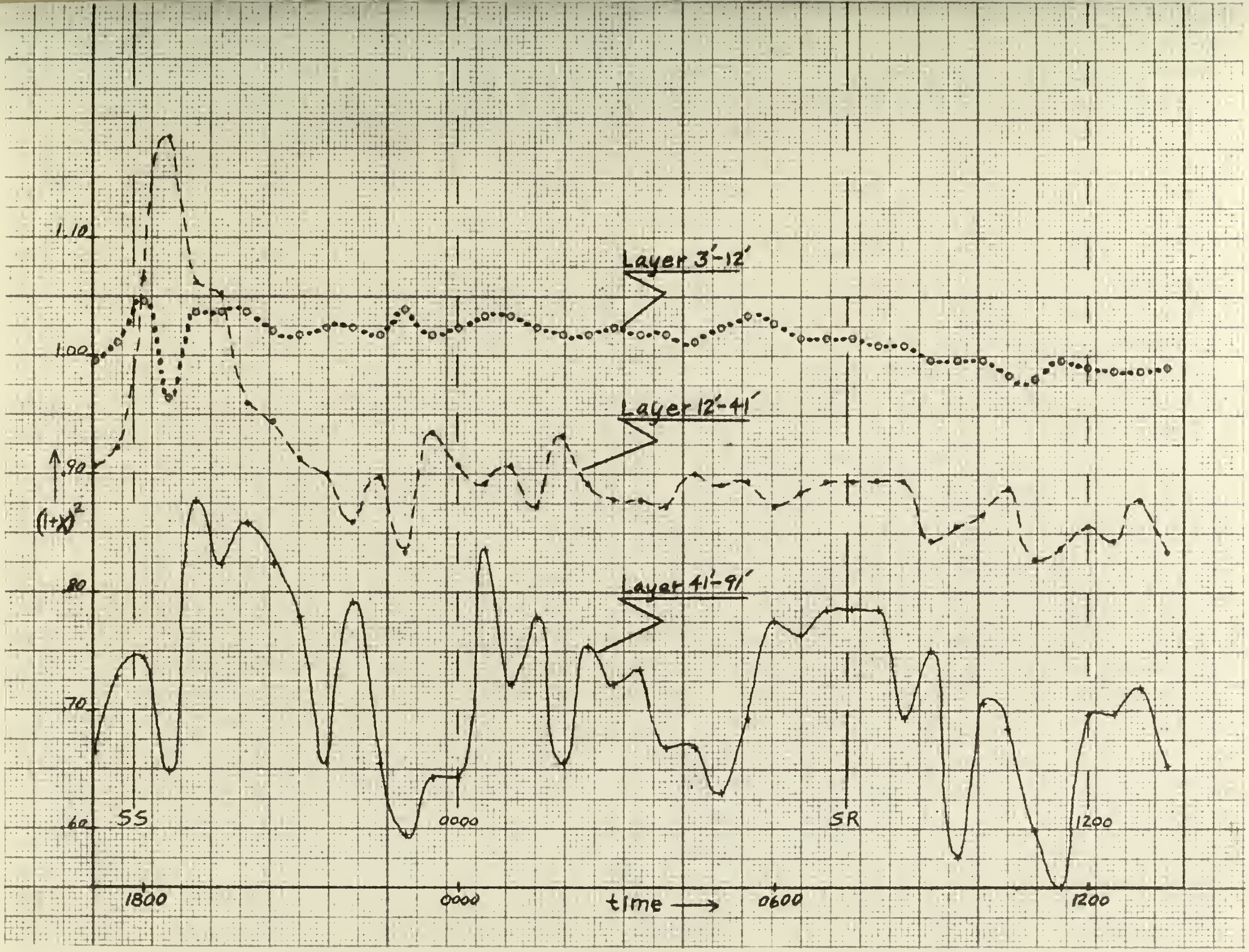

Figure 4. The variation of $(1+x)^2$ with time, for three layers from Run 16.

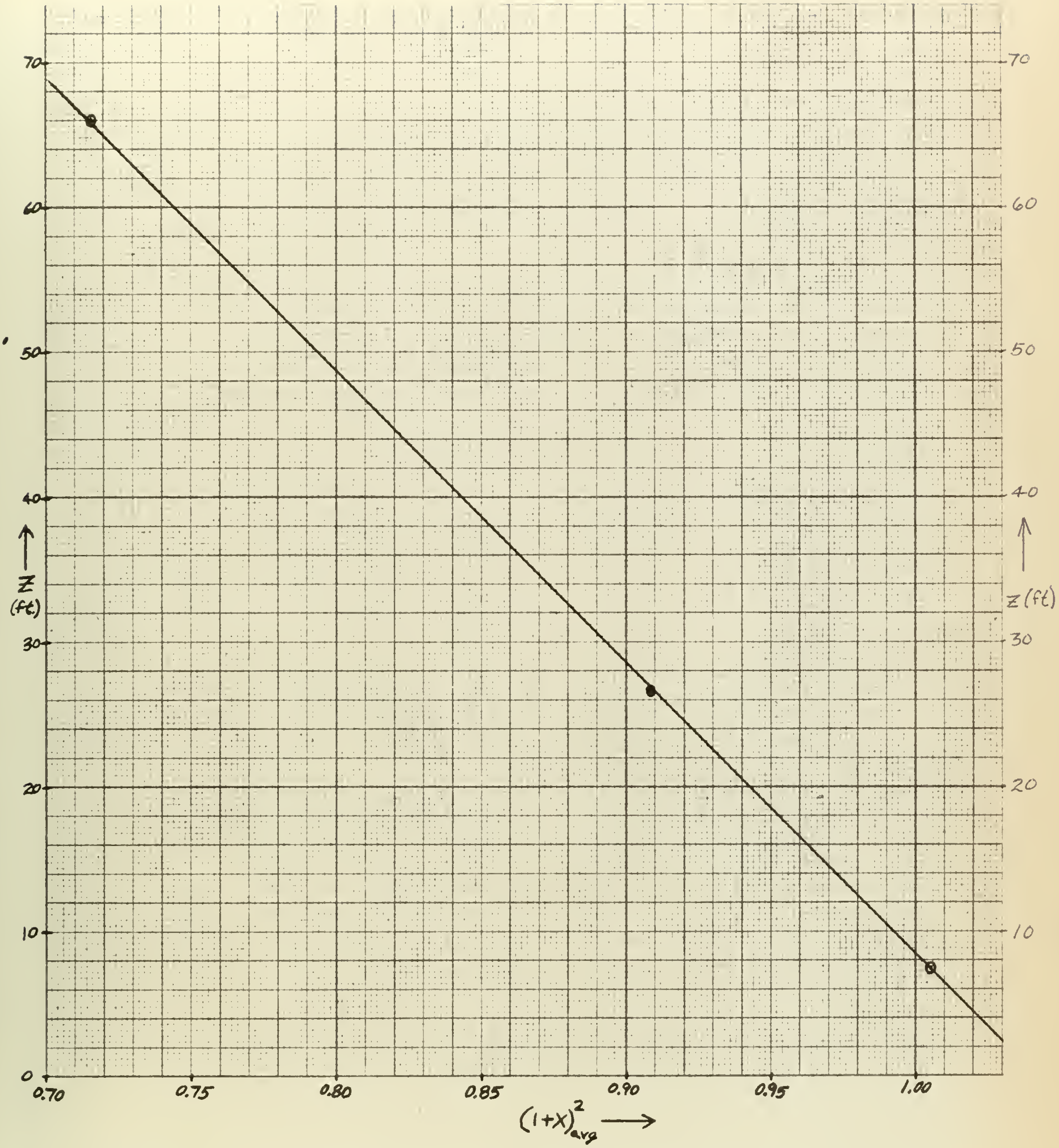

Figure 5. The linear variation of $(1+x)^2_{avg}$ with height for Run 16.

$$(1+X)^2_{avg} = \frac{1}{N}\sum_{i=1}^{N}(1+X)^2_i$$

The average values were then plotted against height, resulting in Fig. 5. The linear decrease of $(1+x)^2_{avg}$ with elevation, for this run, substantiates Deacon's conclusions.

In comparing the values of β obtained in this investigation with those of previous investigators, they were found to fall within the same range. Calder [1] had values of β from .75 to 1.4, Deacon (1948) found that β varied from .75 to 1.2, and Priestly and Sheppard [15] determined the range of β to be from .70 to 1.3. A minimum value of β equal to .84 occurred in this investigation during Run 10, with the maximum value of 1.4 appearing during Run 8.

Investigator	Min β	Max β
Calder	.75	1.4
Deacon	.75	1.2
Priestly & Sheppard	.70	1.3
Johnston	.84	1.4

Table of ranges of β as determined by the indicated investigators.

Table 1

β was found to vary considerably from one observation time to the next, with a maximum variation of around .2 not uncommon for Runs 8 and 10. Longley [11] derived an equation for β that made the usual assumption of z_o constant with changes in stability unnecessary. In this equation β depends only on wind speeds at three levels if the heights are related by $z_3 = z_2 r = z_1 r^2$, where r is any ratio. His equation was

$$\beta = 1 - \frac{\ln \frac{u_3 - u_2}{u_2 - u_1}}{\ln r}$$

Using this equation, he computed values of β from Deacon's data and found an average difference from Deacon's β of from 0.1 to 0.2 depending on the heights selected. He also determined β from other data for various times during both day and night. He found that for daylight hours the range of four values of β taken during the same hour might lay between 0.1 and 0.2, and for inversion conditions the ranges may even be greater. His final conclusions were (1) a possible error of 0.1 or 0.2 does not permit different types of stability to be clearly distinguished in many cases, and (2) it is impossible to calculate β with sufficient accuracy to warrant its use in any practical problems. Deacon, in answering Longley's article, agreed that β is difficult to measure accurately, and said,

> I have never been so optimistic as to suppose that in practical problems it would be profitable to attempt the routine measurements of β in the field.

It should be remembered that Longley's method of measurement was based essentially on curve fitting and had no way of separating out the effects of instability. Longley's conclusions may no longer be tenable since in the present paper, use of $\overline{(1+x)^2}$ accounts for a considerable part of the variability of $\overline{\beta}$ and of u_*.

In perusing the available literature on surface layer turbulence, etc., no investigations into the possible variation of β with height were found. β is generally considered constant with height, at least in the lower 40 feet. However, Deacon (1948) felt there was a possibility that β might vary slightly with elevation, and that if it did, it should slightly increase. Fig. 6 indicates a possible variation of β with height.

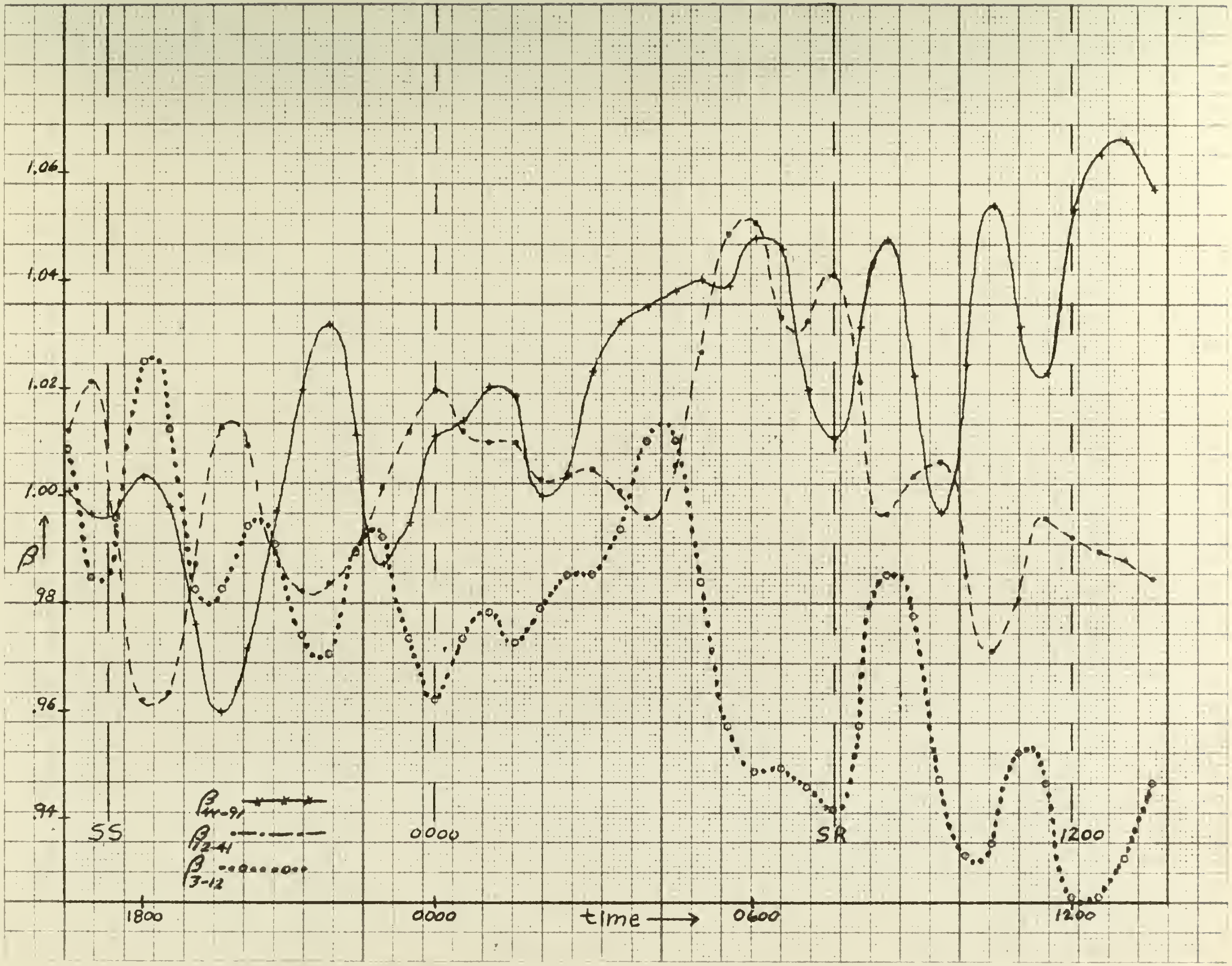

Figure 6. The variation of β with time, for three layers from Run 16.

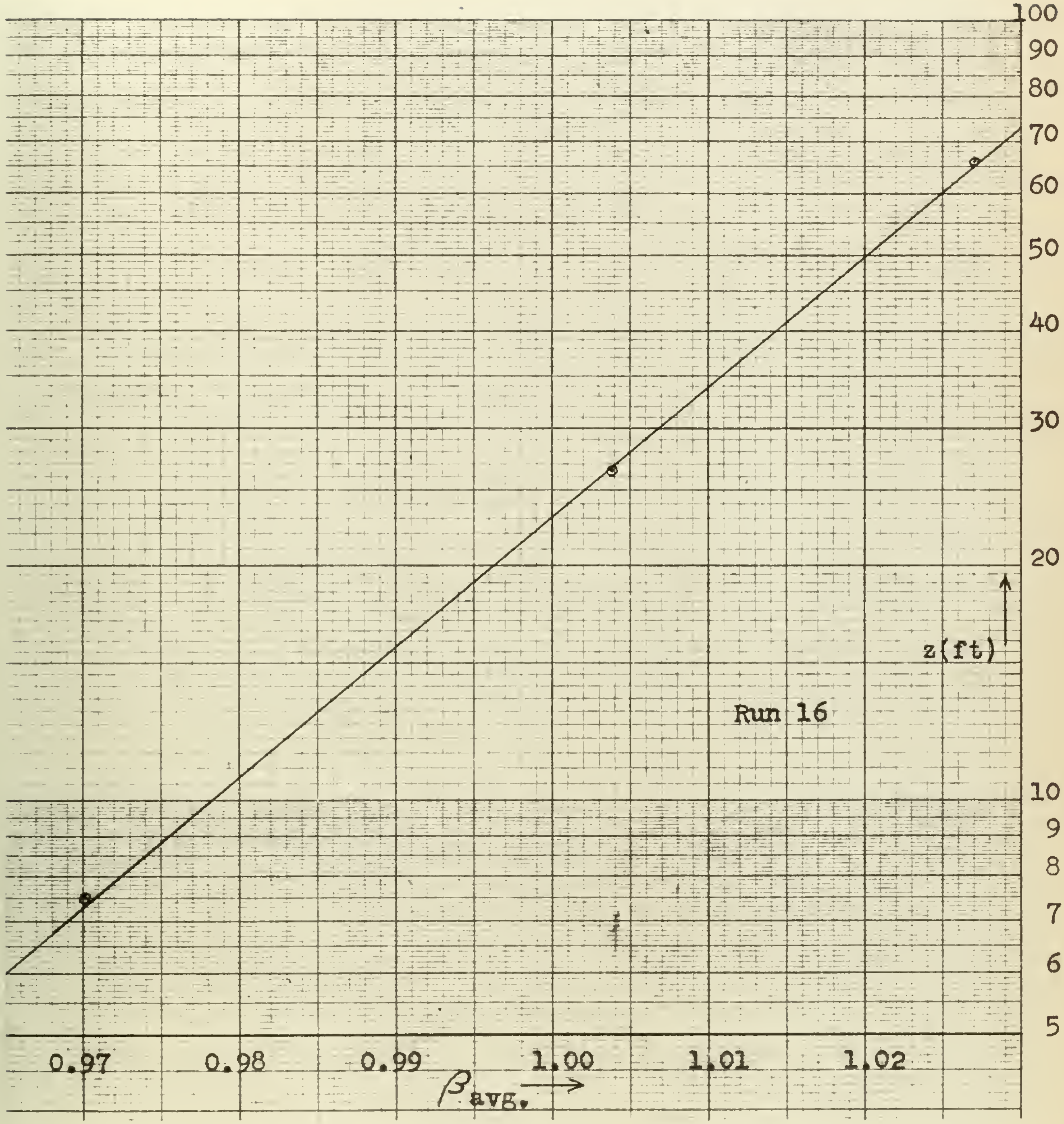

Figure 7. The variation of β with the logarithm of height for Run 16.

β, from Run 16, was averaged for each layer using the same equation as that for $(1+x)^2_{avg}$, with the results as below.

$$\beta_{25'_{avg}} = 0.9702$$

$$\beta_{26.5'_{avg}} = 1.0040$$

$$\beta_{66'_{avg}} = 1.0271$$

It was found that in plotting these values against ln z, as in Fig. 7, the points could be connected by a straight line. This suggests that β increases with the logarithm of height in the friction layer. The average β for each layer was also determined for Run 17. Here it was found that the average β for the upper layer decreased slightly from that for the middle layer. However, a straight line connecting β_{avg} for the middle and lower layer gave a similar slope as that found for Run 16. This could possibly illustrate the fact that Deacon's equation was not applicable to as great a height for Run 17 as for Run 16, due to a difference in thickness of the surface layer, or it could mean that the relationship of with height found from Run 16 was only accidental.

Prior to this paper, there has been no equation for accurately determining the friction velocity for non-neutral conditions without measurements of the surface drag or short period wind velocity fluctuations. Panofsky [14], by making several assumptions, derived an equation for u_* which depends only on the wind speed at three elevations. However, upon comparing his results for u_* with those obtained by measurements of the short period velocity fluctuations, they were found to be less than satisfactory.

Deacon [3], while making an investigation into the variation of the

eddy stress with height, found that values of u_* at 75 feet were much more variable than those at five feet. He attributed this mainly to accelerations associated with large scale eddies. He also found that there seemed to be no systematic variation of u_* with height, although individual measurements showed large departures.

Fig. 8 is the friction velocity from Run 16 for the three layers plotted against time, as determined by Eq. (18). Here it is seen that the fluctuations of u_* for the highest layer are generally of greater amplitude than for the lower two layers. Although this run seems to indicate a possible increase of u_* with height, Run 17 indicated a possible decrease with height. However, there does not seem to be a systematic increase of u_* between the layer 3-12 feet as compared to 12-41 feet. Any possible conclusions that can be made here regarding the variation of u_* in the layer 41-91 feet are tentative, inasmuch as the Deacon profile may not be applicable to these elevations. However there is no clear-cut increase of u_* between the layer 12-41 feet and 41-91 feet.

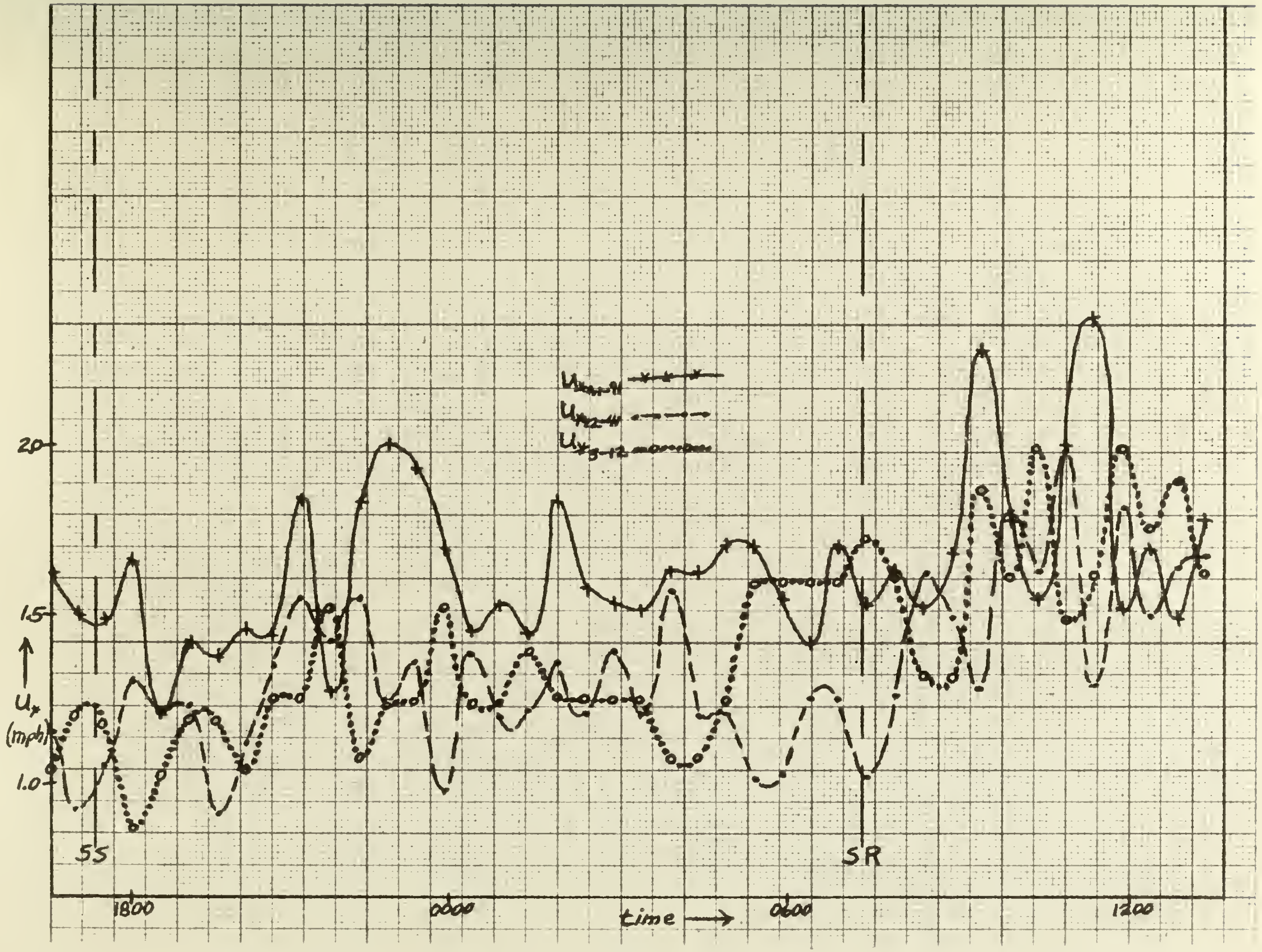

Figure 8. The variation of the friction velocity with time, for three layers from Run 16.

6. Summary and Conclusions

Martin's combination of Lettau's vertical mixing velocity with Deacon's wind profile equation gives values of β and u_* which compare favorably with those of other investigators. However, the equations containing the term $(1+x)^2$ will not apply for $x \leq -1$, which may have the significance of placing an upper limit on the depth of the surface layer. During this investigation x was never found to be less than minus one.

The equation for the friction velocity, Eq. (18), may become an accurate method for determining u_* if it becomes possible to evaluate β with sufficient accuracy. Additional work is needed to compare u_*, as determined from Eq. (18), with that obtained from surface drag measurements or short period wind velocity fluctuations, before an accurate evaluation of the equation can be made.

By comparing a large number of cases it was found possible to use data that was otherwise undesirable because of the inaccuracies of the wind measurements. However, to determine the various parameters for any one observation, it is felt that wind measurements of <u>much</u> greater accuracy are needed.

One of the most important results of this investigation is the determining of a possible variation of β with height and the correlation of β with $(1+x)^2$. However, further work with a larger number of layers is needed to verify this result.

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Appendix I

Run 8

time	$(1+x)^2$	β	u_* (mph)
1300	.528	1.133	.899
1330	.496	1.226	.583
1400	.496	1.226	.583
1430	.497	1.141	.919
1500	.598	1.168	.661
1530	.633	1.161	.648
1600	.708	k.147	.623
1630	.708	1.119	.721
1700	.827	1.163	.489
1730	.911	1.201	.362
1800	1.000	1.275	.223
1830	1.399	1.098	.408
1900	1.684	1.076	.383
1930	2.196	1.007	.422
2000	2.336	.972	.478
2030	2.269	.976	.483
2100	2.487	.964	.468
2130	2.208	1.007	.421
2200	1.879	.976	.481
2230	1.945	1.022	.440
2300	2.078	1.014	.430
2330	1.758	1.007	.527
0000	1.408	1.062	.492
0030	1.638	1.043	.467
0100	1.464	1.030	.562
0130	1.464	1.093	.402

time	$(1+x)^2$	β	U_*
0200	1.408	1.098	.407
0230	1.522	1.088	.396
0300	1.409	1.097	.407
0330	1.145	1.173	.335
0400	1.196	1.118	.431
0430	1.196	1.118	.431
0500	1.249	1.077	.512
0530	1.302	1.157	.320
0600	1.146	1.123	.437
0630	1.125	1.067	.373
0700	.882	1.147	.311
0730	.670	1.275	.257
0800	.409	1.327	.257
0830	.492	1.303	.178
0900	.216	1.405	.320
0930	.277	1.375	.294
1000	.497	1.217	.379
1030	.221	1.403	.318
1100	.347	1.212	.560
1130	.223	1.266	.652
1200	.224	1.266	.652
1230	.089	1.379	.893

Appendix II

Run 10

time	$(1+X)^2$	β	U_* (mph)
1630	.459	1.103	.627
1700	.575	1.081	.565
1730	.574	1.081	.566
1800	2.716	.873	.368
1830	1.112	.956	.575
1900	2.217	.854	.499
1930	2.470	.844	.472
2000	1.552	.989	.344
2030	.032	1.150	1.610
2100	.180	1.124	1.430
2130	2.233	.891	.406
2200	2.488	.881	.384
2230	1.557	.989	.343
2300	2.493	.945	.271
2330	1.359	.937	.520
0000	1.173	.951	.560
0030	1.000	.966	.606
0100	1.361	.937	.520
0130	1.174	1.015	.396
0200	1.440	.918	.704
0230	1.563	.910	.675
0300	1.441	.918	.704
0330	1.324	.963	.599
0400	1.566	.948	.551

Run 10 (contd.)

time	$(1+x)^2$	β	u_*
0430	1.324	.963	.599
0500	1.000	.952	.845
0530	.559	1.043	.922
0600	.486	1.056	.989
0630	1.444	.918	.703
0700	1.325	.926	.734
0730	.722	1.019	.812
0800	.423	1.133	.750
0830	.423	1.069	1.060
0900	.492	1.119	.695
0930	.642	1.095	.608
1000	.165	1.220	1.200
1030	.257	1.201	.856
1100	.368	1.082	1.137
1130	.498	1.054	.977
1200	.048	1.089	4.596

APPENDIX III

Run 16

	Layer 3'-12'			Layer 12'-41'			Layer 41'-91'				
time	$(1+x)^2$	β	u_*(mph)	$(1+x)^2$	β	u_*(mph)	$(1+x)^2$	β	u_*(mph)	$\overline{(1+x)^2}$	$\overline{\beta}$
1630	.992	1.013	1.054	.907	1.009	1.154	.672	1.003	1.633	.782	1.006
1700	1.008	.976	1.207	.921	1.046	.935	.732	.996	1.518	.823	1.010
1730	1.042	.972	1.188	1.060	.995	1.068	.747	.994	1.492	.881	.992
1800	.963	1.065	.875	1.177	.938	1.308	.655	1.005	1.667	.859	.989
1830	1.034	1.008	1.032	1.057	.969	1.235	.877	1.014	1.224	.953	.998
1900	1.034	.973	1.192	1.049	.969	1.239	.826	.964	1.432	.921	.967
1930	1.034	.973	1.192	.960	1.042	.916	.860	.961	1.385	.910	.989
2000	1.017	1.010	1.041	.944	1.006	1.132	.826	.946	1.483	.884	.972
2030	1.015	.985	1.255	.913	.990	1.385	.781	1.030	1.448	.847	1.012
2100	-	-	-	-	-	-	-	-	-	-	-
2130	1.021	.984	1.250	.899	.971	1.560	.663	.988	1.851	.778	.982
2200	1.021	.936	1.531	.859	.996	1.428	.795	1.083	1.283	.839	1.039
2230	1.014	1.020	1.087	.898	.971	1.562	.663	.988	1.851	.776	.985

Run 16 (cont.)

	Layer 3'-12'			Layer 12'-41'			Layer 41'-91'				
time	$(1+x)^2$	β	u_*(mph)	$(1+x)^2$	β	u_*(mph)	$(1+x)^2$	β	u_*(mph)	$\overline{(1+x)^2}$	$\overline{\beta}$
2300	**1.036**	.983	1.242	.835	1.025	1.254	.602	.995	2.010	.723	1.004
2330	1.014	.985	1.255	.933	.988	1.370	.651	.971	1.946	.781	.978
0000	1.021	.936	1.531	.907	1.055	.983	.651	1.044	1.691	.773	1.037
0030	1.029	.984	1.246	.891	.992	1.402	.837	.992	1.454	.874	.991
0100	1.029	.984	1.246	.907	1.017	1.204	.727	1.035	1.539	.817	1.024
0130	1.021	.958	1.398	.873	1.021	1.226	.781	1.030	1.448	.836	1.019
0200	1.014	.985	1.255	.931	.988	1.372	.663	.988	1.851	.787	.988
0230	1.014	.985	1.255	.890	1.019	1.214	.759	1.000	1.579	.829	1.005
0300	1.021	.984	1.250	.878	.994	1.412	.728	1.035	1.538	.807	1.016
0330	1.014	.985	1.255	.880	1.020	1.222	.741	1.034	1.515	.815	1.024
0400	1.014	1.020	1.087	.872	.974	1.585	.676	1.041	1.638	.775	1.017
0430	1.007	1.021	1.090	.899	1.018	1.208	.676	1.041	1.638	.783	1.031
0500	1.021	.984	1.250	.892	1.019	1.213	.638	1.046	1.719	.761	1.031
0530	1.030	.945	1.590	.894	1.064	1.032	.670	1.045	1.706	.797	1.041

Run 16 (cont.)

	Layer 3'-12'			Layer 12'-41'			Layer 41'-91'				
time	$(1+x)^2$	β	u_*(mph)	$(1+x)^2$	β	u_*(mph)	$(1+x)^2$	β	u_*(mph)	$(1+x)^2$	β
0600	1.024	.945	1.595	.872	1.066	1.045	.778	1.037	1.559	.834	1.037
0630	1.012	.948	1.604	.886	1.027	1.270	.767	1.093	1.419	.831	1.056
0700	1.012	.948	1.604	.893	1.026	1.265	.790	.980	1.713	.847	.992
0730	1.012	.928	1.733	.895	1.064	1.031	.790	1.035	1.539	..847	1.034'
0800	1.006	.947	1.609	.895	1.026	1.263	.790	1.003	1.639	.847	1.005
0830	1.006	.996	1.314	.895	.979	1.631	.670	1.100	1.533	.795	1.049
0900	.994	.998	1.321	.845	1.005	1.501	.756	1.007	1.701	.809	1.005
0930	.994	.914	1.869	.857	1.030	1.291	.586	.986	2.283	.717	.993
1000	.994	.949	1.619	.866	.966	1.816	.712	1.011	1.790	.791	.989
1030	.982	.901	1.994	.888	.980	1.637	.690	1.101	1.551	.785	1.041
1100	.973	.973	1.491	.829	.955	2.005	.607	1.024	2.047	.718	.996
1130	.994	.949	1.619	.837	1.032	1.306	.558	.990	2.378	.695	1.000
1200	.988	.901	1.988	.856	.966	1.827	.702	1.099	1.529	.781	1.035
1230	.982	.932	1.759	.843	1.005	1.502	.703	1.045	1.700	.778	1.020
1300	.982	.916	1.880	.879	.980	1.645	.724	1.097	1.489	.802	1.040
1330	.988	.950	1.623	.835	.986	1.688	.660	1.050	1.792	.751	1.018

APPENDIX IV

Run 17

	Layer 3'-12'			Layer 12'-41'			Layer 41'-91'				
time	$(1+x)^2$	β	u_*(mph)	$(1+x)^2$	β	u_*(mph)	$(1+x)^2$	β	u_*(mph)	$\overline{(1+x)^2}$	$\overline{\beta}$
1630	.988	.928	1.365	.594	.987	1.556	.697	1.014	1.292	.693	.997
1700	.988	.928	1.365	.743	1.040	1.222	.842	.929	1.212	.824	.965
1730	1.024	.925	1.341	.867	.956	1.076	.841	.975	1.134	.868	.964
1800	1.042	.940	1.213	1.023	.946	.915	1.000	.931	.928	1.012	.967
1830	1.031	.989	.944	1.019	.985	.908	.919	.996	1.007	.963	.992
1900	1.034	.941	1.218	1.009	.947	.928	.918	1.043	.942	.960	1.001
1930	1.023	.963	1.095	.959	.951	.975	.998	.936	1.021	.988	.940
2000	1.024	.942	1.224	1.000	.987	.925	.840	.975	1.135	.911	.976
2030	1.024	.963	1.094	.954	.990	.968	.840	.956	1.167	.896	.968
2100	1.012	.964	1.101	.977	.967	.952	.918	1.043	.942	.947	1.009
2130	1.018	.911	1.452	.954	.968	.974	.918	.950	1.077	.940	.952
2200	1.033	.909	1.442	.991	1.018	.923	.983	1.038	.885	.991	1.019
2230	1.024	.910	1.448	.945	.969	.984	.950	.933	1.067	.956	.943
2300	1.028	.974	1.162	.925	1.002	1.125	1.013	.998	1.031	.986	.997

Run 17 (Cont.)

	Layer 3'-12'			Layer 12'-41'			Layer 41'-91'				
time	$(1+x)^2$	β	u_*(mph)	$(1+x)^2$	β	u_*(mph)	$(1+x)^2$	β	u_*(mph)	$(1+x)^2$	β
2330	1.029	.974	1.162	.925	1.033	1.114	1.176	.915	1.002	1.079	.959
0000	1.020	.954	1.305	.961	1.030	1.074	1.205	.959	.917	1.105	.982
0030	1.026	.936	1.425	1.007	1.026	1.025	1.205	.940	.943	1.121	.968
0100	1.024	.954	1.302	1.055	1.023	.980	1.162	.961	.947	1.113	.981
0130	1.028	.921	1.537	1.036	.993	1.007	1.026	.997	1.019	1.029	.988
0200	;.023	.975	1.165	1.092	.989	.957	.936	1.004	1.107	.997	.996
0230	1.028	1.001	1.006	1.111	.966	.948	1.066	.968	1.024	1.077	.970
0300	1.028	.953	1.299	1.074	1.022	.964	1.204	.960	.917	1.143	.979
0330	1.026	.953	1.301	1.081	.968	.974	1.204	.987	.882	1.145	.977
0400	1.028	.953	1.299	1.111	.966	.948	1.013	.998	1.031	1.047	.980
0430	1.038	.952	1.293	1.112	1.019	.931	.999	.972	1.086	1.040	.985
0500	1.028	1.039	.822	1.037	.972	1.014	1.108	.965	.990	1.076	.975
0530	1.044	.973	1.153	1.085	.968	.970	.987	1.000	1.056	.987	1.025
0600	1.038	.973	1.157	1.074	.969	.980	1.013	1.045	.964	1.012	1.036
0630	1.055	.972	1.147	.971	.959	1.087	1.066	.995	.985	.981	1.034

Run 17 (cont.)

	Layer 3'-12'			Layer 12-41'			Layer 41'-91'				
time	$(1+x)^2$	β	u_*(mph)	$(1+x)^2$	β	u_*(mph)	$(1+x)^2$	β	u_*(mph)	$\overline{(1+x)^2}$	$\overline{\beta}$
0700	1.054	.951	1.283	.975	.976	1.077	1.205	1.033	.824	1.006	1.114
0730	1.034	.959	1.343	1.057	1.028	1.050	1.011	.976	1.147	.992	1.029
0800	1.037	.980	1.200	.954	.983	1.181	1.116	.997	1.009	.991	1.054
0830	1.020	.982	1.210	.926	1.007	1.206	.989	1.051	1.052	1.030	.971
0900	1.049	.941	1.461	.850	1.014	1.312	1.057	.973	1.102	.983	.988
0930	.984	.964	1.377	.794	1.050	1.388	.945	.981	1.219	1.002	.899
1000	.980	.947	1.512	.836	.976	1.349	.740	.978	1.563	.974	.796
1030	.984	.947	1.509	.969	.965	1.169	.989	.958	1.203	.959	.981
1100	.960	.966	1.394	.851	.961	1.333	1.057	1.047	.991	1.010	.979
1130	.968	.966	1.388	.822	.995	1.364	1.364	1.030	.787	1.145	1.012
1200	.967	.949	1.522	.800	1.019	1.391	.891	1.058	1.155	.869	1.034
1230	.957	.967	1.397	.801	.979	1.394	.694	1.028	1.549	.759	1.006
1300	.964	.922	1.760	.895	1.041	1.235	.741	1.024	1.460	.815	1.10
1330	.941	.968	1.408	.809	1.101	1.339	.840	1.015	1.303	.840	1.039
1400	.972	.948	1.518	.782	.998	1.433	.625	1.104	1.710	.711	1.021
1430	.930	.969	1.417	.880	1.011	1.267	.522	1.094	1.873	.682	1.054

Run 17 (cont.)

	Layer 3'-12'			Layer 12'-41'			Layer 41'-91'				
time	$(1+x)^2$	β	u_*(mph)	$(1+x)^2$	β	u_*(mph)	$(1+x)^2$	β	u_*(mph)	$(1+x)^2$	β
1500	.930	.990	1.267	.852	1.014	1.309	.742	.997	1.517	.797	1.012
1530	.961	.935	1.649	.716	1.027	1.551	.892	.991	1.227	.841	1.016
1600	.969	.987	1.241	.852	1.045	1.295	.695	1.028	1.547	.775	1.029
1630	.976	1.013	1.071	.909	1.040	1.215	.892	1.012	1.234	.907	1.021
1700	.999	.984	1.222	1.003	1.032	1.105	.892	1.012	1.234	.940	1.016
1730	1.051	.971	.857	1.134	.995	.682	.977	.981	.813	1.036	.984
1800	1.121	.965	.830	1.134	.964	.689	1.221	.966	.665	1.182	.965
1830	1.176	.999	.662	1.135	.964	.689	1.523	.924	.566	1.360	.945
1900	1.104	.939	.966	.968	.976	.805	1.670	.945	.501	1.381	.955
1930	1.232	.994	.646	.814	.989	.953	2.089	.930	.409	1.581	.956
2000	1.176	.933	.936	.699	1.031	1.093	1.383	.896	.649	1.136	.945
2030	1.113	.918	1.076	.843	.986	.920	1.274	1.009	.598	1.116	.992
2100	1.158	.914	1.054	.755	1.026	1.015	1.734	.881	.529	1.352	.932
2130	1.140	.898	1.164	.699	1.001	1.105	1.525	.924	.566	1.213	.947
2200	1.140	.936	.950	.755	.973	1.032	1.672	.917	.520	1.316	.938
2230	1.140	.963	.823	.699	1.031	1.094	1.989	.933	.428	1.477	.969
2300	1.104	.966	.836	.755	.973	1.033	2.159	.974	.371	1.589	.973